E2.5 Signals & Linear Systems

Tutorial Sheet 5 – Laplace Transform & Frequency Response

(Lectures 7 - 9)

1.* Using Laplace transform, solve the following differential equations:

a) $(D^2+3D+2)y(t)=Df(t)$ if $y(0^-)=\dot{y}(0^-)=0$ and $f(t)=u(t)$

b) $(D^2+4D+4)y(t)=(D+1)f(t)$ if $y(0^-)=2, \dot{y}(0^-)=1$ and $f(t)=e^{-t}u(t)$

c) $(D^2+6D+25)y(t)=(D+2)f(t)$ if $y(0^-)=\dot{y}(0^-)=1$ and $f(t)=25u(t)$.

2.* For each of the system described by the following differential equations, find the system transfer function.

a) $\dfrac{d^2y}{dt^2}+11\dfrac{dy}{dt}+24y(t)=5\dfrac{df}{dt}+3f(t)$

b) $\dfrac{d^3y}{dt^3}+6\dfrac{d^2y}{dt^2}-11\dfrac{dy}{dt}+6y(t)=3\dfrac{d^2f}{dt^2}+7\dfrac{df}{dt}+5f(t)$

c) $\dfrac{d^4y}{dt^4}+4\dfrac{dy}{dt}=3\dfrac{df}{dt}+2f(t)$.

3.** For a system with transfer function

$$H(s)=\frac{s+5}{s^2+5s+6}$$

a) Find the zero-state response if the input *f(t)* is

(i) $e^{-4t}u(t)$ (ii) $e^{-3t}u(t)$ (iii) $e^{-4(t-5)}u(t-5)$

b) For this system write the differential equation relating the output *y(t)* to the input *f(t)*.

4.** For the circuit shown in Figure Q4, the switch is in open position for a long time before t = 0, when it is closed instantaneously.

a) Write loop equations in time domain for t ≥ 0.

b) Solve for $y_1(t)$ and $y_2(t)$ by taking the Laplace transform of loop equations found in part a).

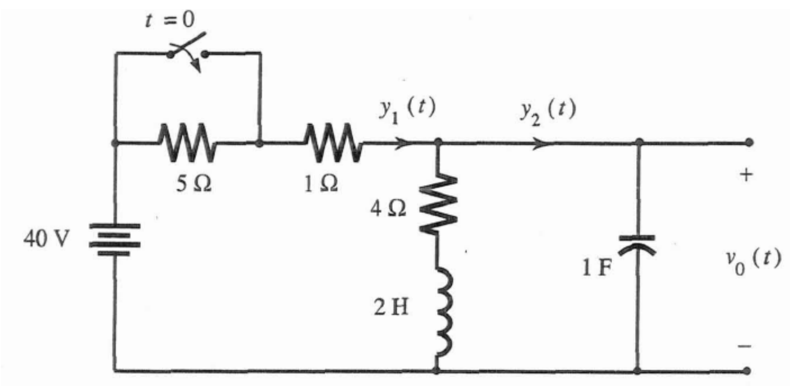

Fig. Q4

5.** The switch in the circuit of Fig. Q5 is closed for a long time and then opened instantaneously at t = 0. Find and sketch the current *y(t)*.

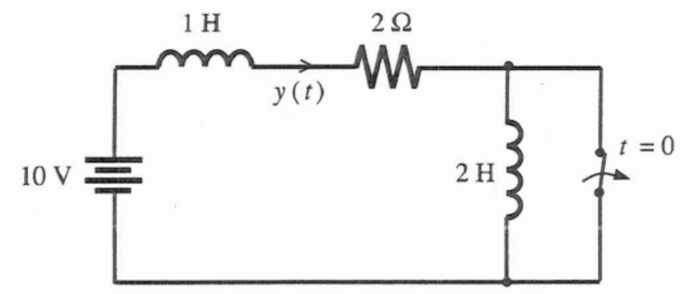

Fig. Q5

6.** For the second-order op amp circuit shown in Fig. Q6, show that the transfer function *H(s)* relating the output voltage $v_o(t)$ to the input voltage *f(t)* is given by

$$H(s) = \frac{-s}{s^2 + 8s + 12}.$$

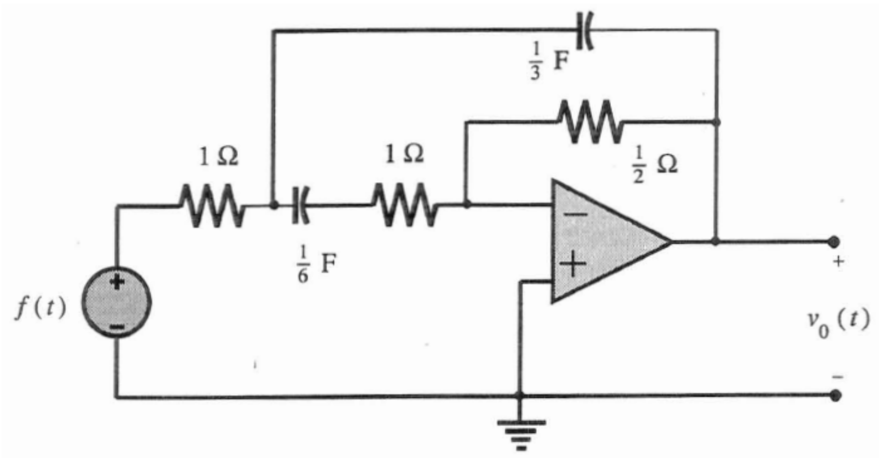

Fig. Q6

7.* Using the initial and final value theorems, find the initial and final values of the zero-state response of a system with the transfer function

$$H(s) = \frac{6s^2 + 3s + 10}{2s^2 + 6s + 5}$$

and the input is

a) $u(t)$

b) $e^{-t}u(t)$.

8.** For a LTI system described by the transfer function

$$H(s) = \frac{s+3}{(s+2)^2}$$

Find the system response to the following inputs:

a) $\cos(2t + 60^\circ)$

b) $\sin(3t - 45^\circ)$

c) e^{j3t}

9.** Using graphical method, draw a rough sketch of the amplitude and phase response of the LTI system described by the transfer function

$$H(s)=\frac{s^2-2s+50}{s^2+2s+50}=\frac{(s-1-j7)(s-1+j7)}{(s+1-j7)(s+1+j7)}.$$

10.*** Using graphical method, draw a rough sketch of the amplitude and phase response of LTI systems whose pole-zero plots are shown in Fig. Q10(a) & (b).

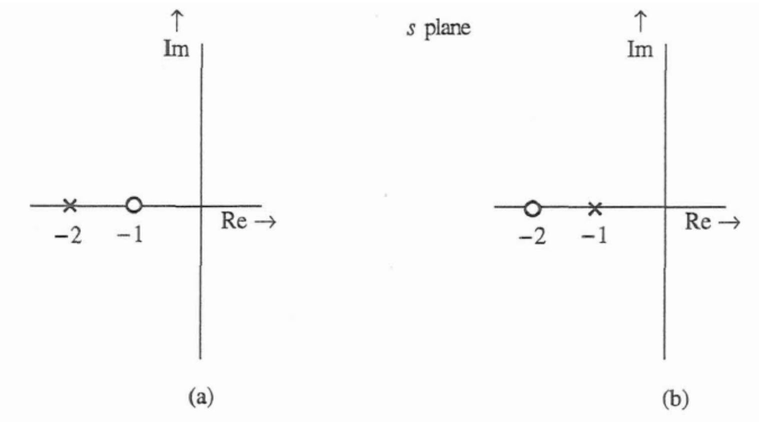

Fig. Q10